Fast Algorithms for Model-Based Smoothing Of Discontinuous Signals and Images

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Extended Abstract

Discontinuities in functions frequently encode significant information: for instance, they represent the boundaries of cellular structures in microscopic images, they correspond to change points in microarray data, and they define tissue layers in tomographic images. Since classical smoothing methods destroy this important information, discontinuity preserving models such as the Potts model, the Mumford-Shah model and the Blake-Zisserman models have been developed. Such free-discontinuity problems are algorithmically challenging as they lead to nonsmooth and nonconvex problems. In the talk, we start discussing the one-dimensional case and we look at recent advances for solving these problems efficiently, exactly and numerically stable [1,2]. We then turn to the higher dimensional case, where only approximative solutions are possible. We study splitting approaches based on the alternating method of multipliers (ADMM) [3] or on iterative majorization minimization [4]. For the higher-order models, where smoothing is based on higher-order derivatives, we study a splitting approach based on Taylor jets [5].

References

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